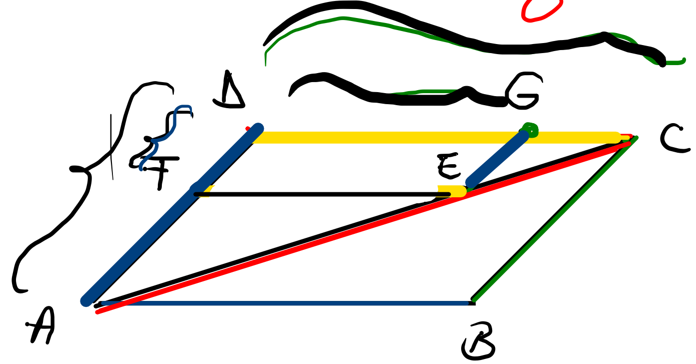
Asemănarea triunghiurilor



1) ABCD par

$E \in (AC)$, $F \in (AD)$

$G \in (CD)$

$EF \parallel AB$ ✓

$\boxed{EG \parallel BC}$ ✓

$$\frac{FD}{AD} + \frac{DG}{DC} = 1$$

ABCDpar $\Rightarrow$ $\left.\begin{array}{l} AB \parallel DC \\ EF \parallel AB \end{array}\right\} \Rightarrow EF \parallel DC$

$\left.\begin{array}{l} BC \parallel AD \\ EG \parallel BC \end{array}\right\} \Rightarrow AD \parallel EG$

$\triangle DAC$: $FE \parallel DC \xrightarrow{TTh} \boxed{\frac{FD}{AD}} = \boxed{\frac{EC}{AC}}$

$\triangle ADC$: $AD \parallel EG \xrightarrow{TTh} \frac{DG}{DC} = \frac{AE}{AC}$

$$\frac{FD}{AD} + \frac{DG}{DC} = \frac{EC}{AC} + \frac{AE}{AC} = \frac{EC + AE}{AC} = \frac{\cancel{AC}}{\cancel{AC}} = 1$$

②

$\triangle ABC$

$D \in (AC)$

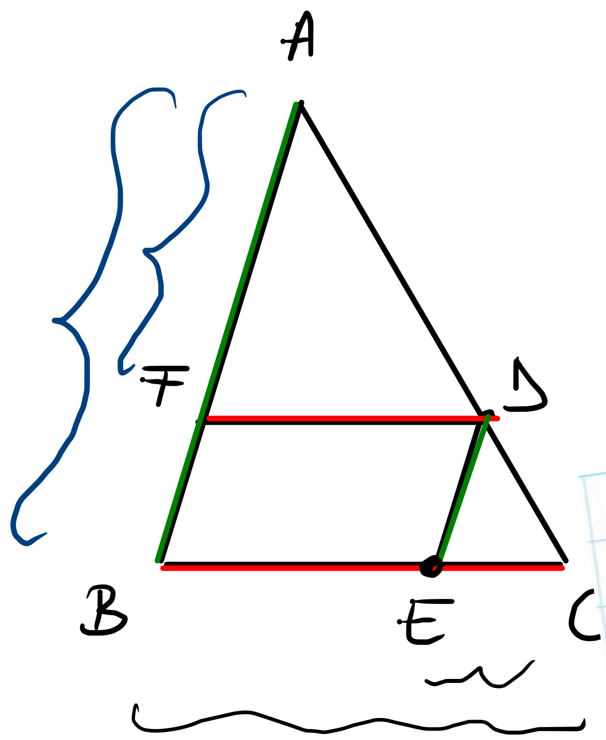
$DE \parallel AB$

$E \in (BC)$

$DF \parallel BC$

$F \in AB$.

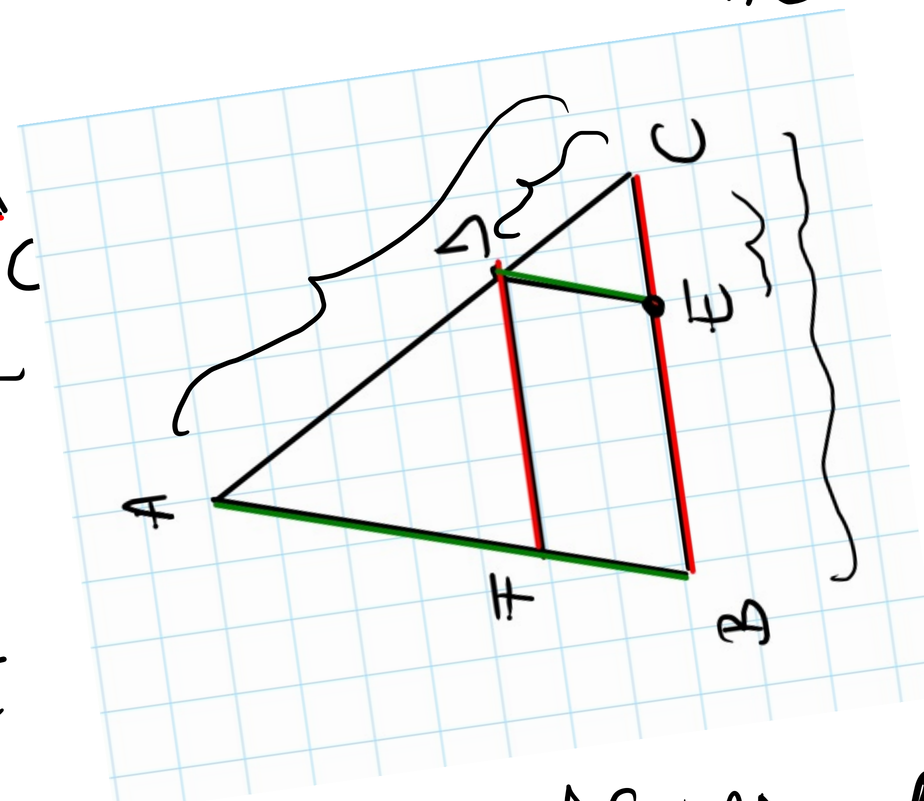
$\frac{CE}{BC} + \frac{AF}{AB} = 1$



$\triangle ABC$: $FD \parallel BC \xrightarrow{TTh} \frac{AF}{AB} = \frac{AD}{AC}$

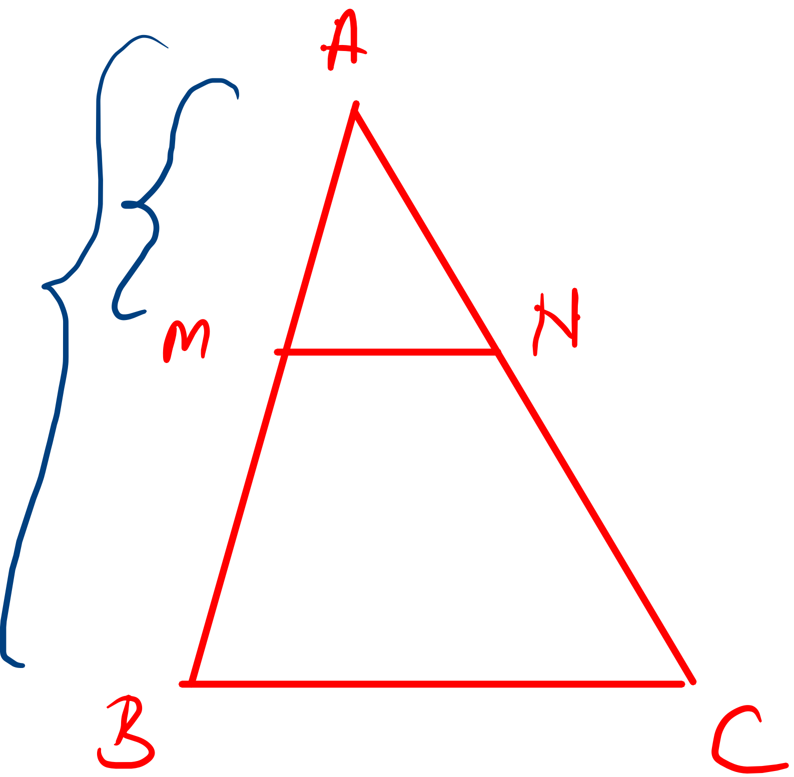
$\triangle ABC$: $ED \parallel AB \xrightarrow{TTh}$

$\frac{CE}{BC} = \frac{DC}{AC}$



$\frac{CE}{BC} + \frac{AF}{AB} = \frac{DC}{AC} + \frac{AD}{AC} = \frac{DC + AD}{AC} = \frac{AC}{AC} = 1$

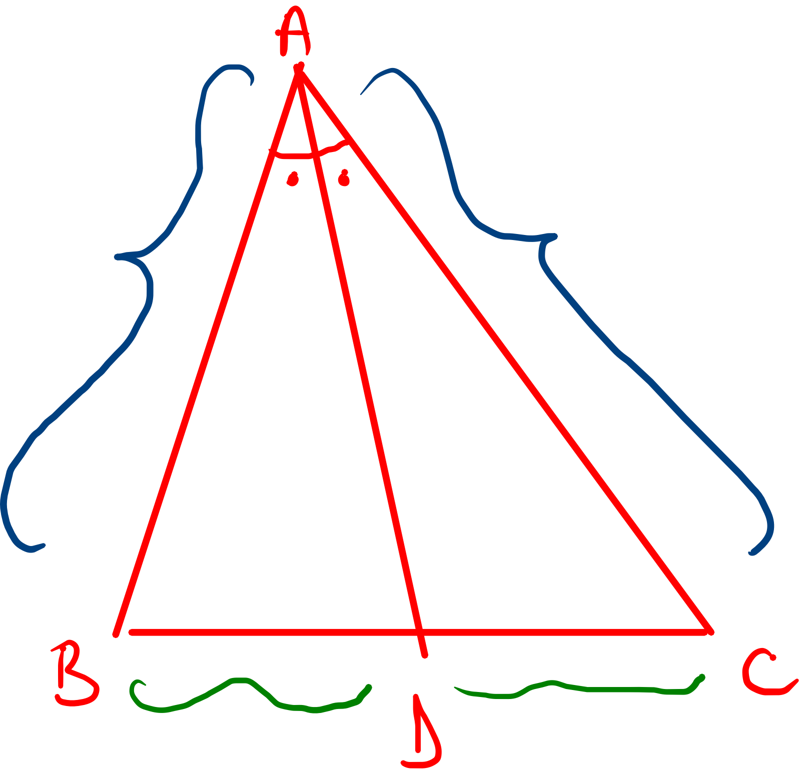
Teorema lui Thales



$$\left.\begin{array}{l}\triangle ABC: \\ MN \parallel BC\end{array}\right\} \xRightarrow{TTh} \begin{cases} \dfrac{AM}{AB} = \dfrac{AN}{AC} \\[2ex] \dfrac{AM}{MB} = \dfrac{AN}{NC} \\[2ex] \dfrac{MB}{AB} = \dfrac{NC}{AC} \end{cases}$$

Ac.

Teorema bisectoarei



$\triangle ABC$: $[AD$ bis $\sphericalangle BAC$ $\xRightarrow{T\,bis}$ $\boxed{\dfrac{AB}{AC} = \dfrac{BD}{DC}}$

$$\frac{AC}{AB} = \frac{DC}{BD}$$

$$\frac{AB}{BD} = \frac{AC}{DC}$$

③

$\triangle ABC$

M - mij BC

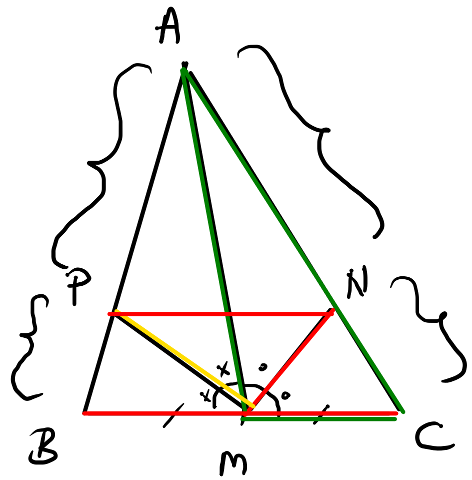
MN - bis $\sphericalangle AMC$

$N \in (AC)$

MP - bis $\sphericalangle AMB$

$P \in (AB)$

$PN \parallel BC$



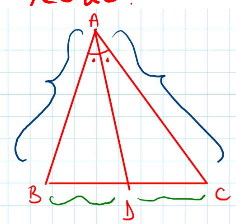
M - mij $BC \Rightarrow \boxed{BM \equiv MC}$

$\triangle AMC$: $[MN$ bis $\sphericalangle AMC \xRightarrow{T bis} \boxed{\dfrac{AM}{MC}} = \dfrac{AN}{NC}$

$\triangle ABM$: $[MP$ bis $\sphericalangle AMB \Rightarrow \boxed{\dfrac{AM}{MB}} = \dfrac{AP}{PB}$

$$\frac{AN}{NC} = \frac{AP}{PB} \xRightarrow{RTTh} PN \parallel BC$$

Teorie:

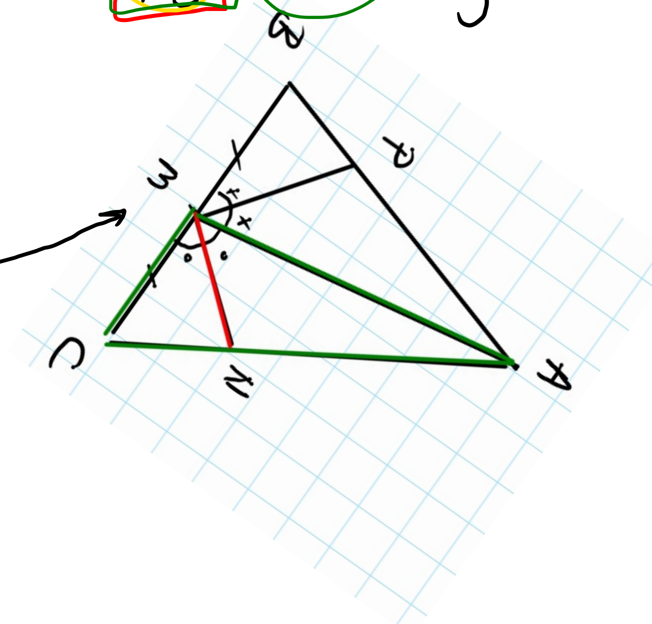


$\triangle ABC$: $[AD$ bis $\sphericalangle BAC$

$\xRightarrow{T bis} \boxed{\dfrac{AB}{AC} = \dfrac{BD}{DC}}$

$\dfrac{AC}{AB} = \dfrac{DC}{BD}$

$AB \quad AC$



④

$\triangle MNP$

$[MQ$ bis $\sphericalangle NMP$

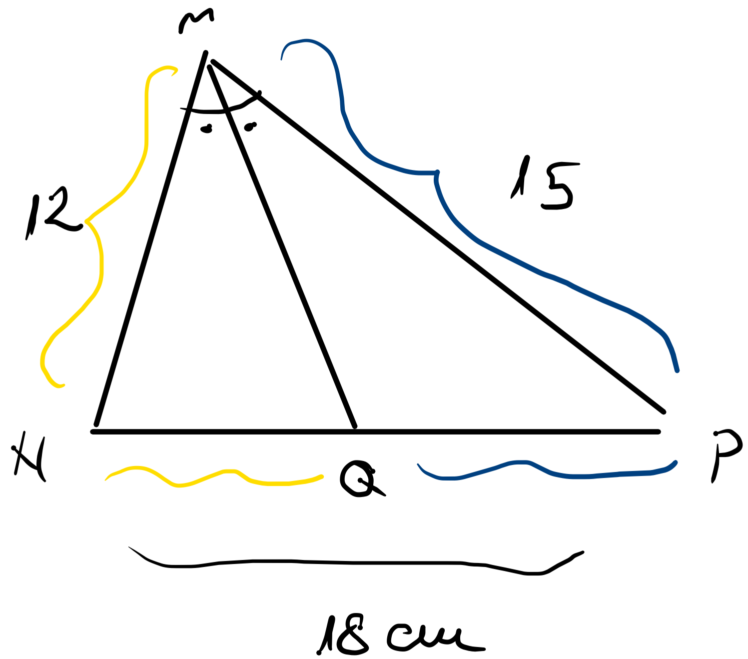
$Q \in NP$

$MN = 12$ cm.

$MP = 15$ cm.

$NP = 18$ cm

$NQ, QP = ?$



$\triangle MNP$: $[MQ$ bis $\sphericalangle NMP \xRightarrow{T\,bis}$

$$\frac{MN}{MP} = \frac{NQ}{QP}$$

$$\frac{12}{15}^{\,3} = \frac{NQ}{QP}$$

$$\boxed{\frac{4}{5} = \frac{NQ}{QP}}$$

$$\Rightarrow \frac{NQ}{4} = \frac{QP}{5} = k$$

$$\left.\begin{array}{l} NQ = 4k \\ QP = 5k \end{array}\right\} \begin{array}{l} NQ + QP = NP \\ 4k + 5k = NP \end{array}$$

$NP = 9k = 18\,cm \Rightarrow k = 2\,cm$

$NQ = 4 \cdot 2 = 8\,cm$

$QP = 5 \cdot 2 = 10\,cm.$

5. $\triangle MNP$ is

$MN \equiv MP$ ✓

$MN = 18$ cm

NQ - mediană

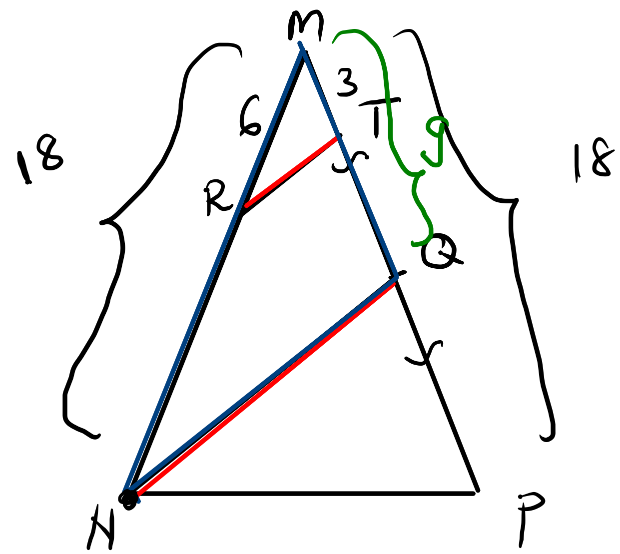
$Q \in MP$, $R \in MN$

$T \in MP$

$MR = 6$ cm

$MT = 3$ cm.

$RT \parallel NQ$



Mediana este segmentul care unește vârful de mijlocul laturii opuse.

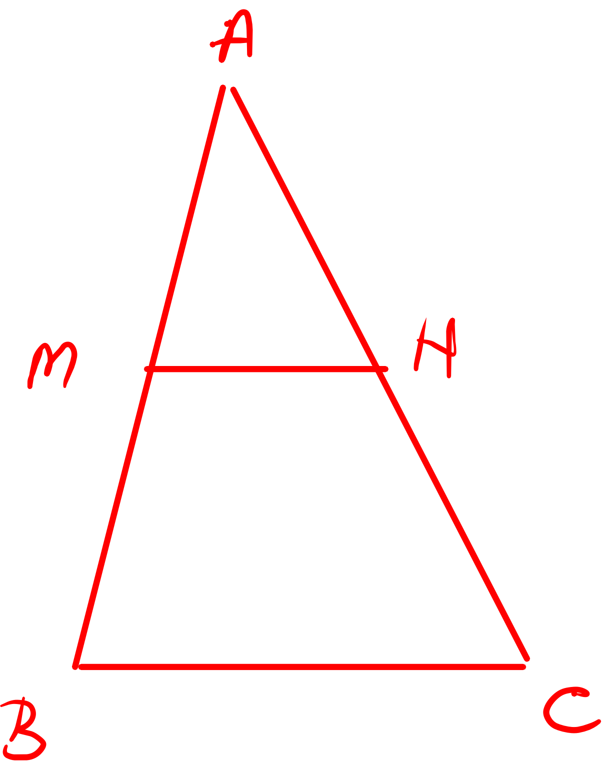
$MN = MP = 18$ cm

NQ - mediană $\underset{\triangle MNP}{}$ $\Rightarrow Q$ - mij $MP \Rightarrow MQ = QP = \frac{MP}{2} = \frac{18}{2} = 9$ cm

$\triangle MQN$:

$$\left.\begin{aligned} \frac{MR}{MN} &= \frac{6}{18} = \frac{1}{3} \\ \frac{MT}{MQ} &= \frac{3}{9} = \frac{1}{3} \end{aligned}\right\} \frac{MR}{MN} = \frac{MT}{MQ} \xRightarrow{RTTh} RT \parallel NQ$$

Teorema fundamentală a asemănării
(TFA)



$$\triangle ABC: \quad MN \parallel BC \xRightarrow{TFA} \triangle AMN \sim \triangle ABC$$

$$\Rightarrow \frac{AM}{AB} = \frac{AN}{AC} = \frac{MN}{BC}$$

6

$\triangle MNP$

$MN = 16$ cm

$NP = 32$ cm

$MP = 40$ cm

$T \in MN$

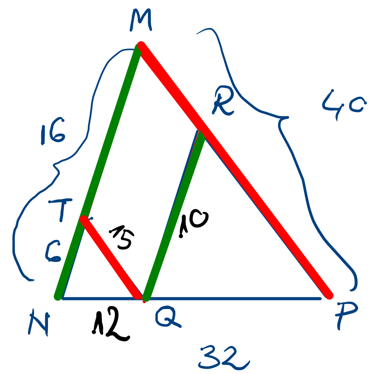
$TN = 6$ cm

$TQ \parallel MP$, $Q \in (NP)$

$QR \parallel MN$, $R \in (MP)$

a) MTQR natura

b) $P_{MTQR} = ?$



$TQ \parallel MP \Rightarrow TQ \parallel MR$

$QR \parallel MN \Rightarrow QR \parallel MT$ } MRQT par.

$P_{MTQR} = 2 \cdot RQ + 2 \cdot TQ = 2 \cdot 10 + 2 \cdot 15 = 50$ cm.

$\triangle MNP$: $QR \parallel MN \xrightarrow{TFA} \triangle RQP \sim \triangle MNP$

$\Rightarrow \frac{RQ}{MN} = \frac{RP}{MP} = \frac{QP}{NP}$

$\Rightarrow \frac{RQ}{16} = \frac{RP}{40} = \frac{QP^{20}}{32} \Rightarrow RQ = \frac{\cancel{16}^{1} \cdot \cancel{20}^{10}}{\cancel{32}_{\cancel{2}}} = 10$

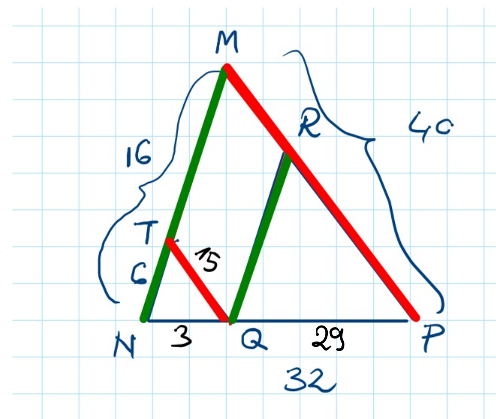
$\triangle MNP$: $QT \parallel MP \xrightarrow{TFA} \triangle TNQ \sim \triangle MNP \Rightarrow$

$\Rightarrow \frac{TN}{MN} = \frac{TQ}{MP} = \frac{NQ}{NP}$

$\Rightarrow \frac{6}{16} = \frac{TQ}{40} = \frac{NQ}{32}$

$TQ = \frac{{}^{3}\cancel{6} \cdot \cancel{40}^{\cancel{10}^{5}}}{\cancel{16}_{\cancel{4}_{2}}} = 15$ cm

$NQ = \frac{{}^{3}\cancel{6} \cdot \cancel{32}^{2}}{\cancel{16}_{\cancel{8}}} = 12$



$QP = NP - NQ = 32 - 12 = 20$ cm.